MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

ECT1026 – FIELD THEORY

(All sections / groups)

8 OCTOBER 2015
2:30 p.m. – 4:30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of eight (8) pages including this cover page with four (4) questions only.

2. Attempt all **FOUR (4)** questions. The distribution of the marks for each question is given.

3. Please write all your answers clearly in the answer booklet provided.

Question 1

a) Magnetic circuits can be analyzed (by analogy) using similar techniques as in electrical circuits. Write down the analogous electrical quantities of the following magnetic quantities:

i) Magneto-motive force, $\mathfrak{F}$. [1 mark]
ii) Magnetic flux, Φ. [1 mark]
iii) Reluctance, $\mathfrak{R}$. [1 mark]

b) Give three differences between electric and magnetic circuits. [3 marks]

c) Briefly explain the phenomenon of *eddy* current loss and the method to reduce it. [3 marks]

d) In the iron core shown in Figure Q1(d), the coil F_1 is supplying 1000 AT in the direction indicated. The relative permeability of iron may be taken as 2500. Neglect fringing effect and flux leakage.

i) Draw the equivalent circuit of the iron core. [4 marks]
ii) Calculate reluctance R_{bafe}, R_{be} and R_{bcde}. [5 marks]
iii) Find the mmf of coil F_2 and its current direction to produce an air-gap flux Φ of 5 mWb in the direction shown. [7 marks]

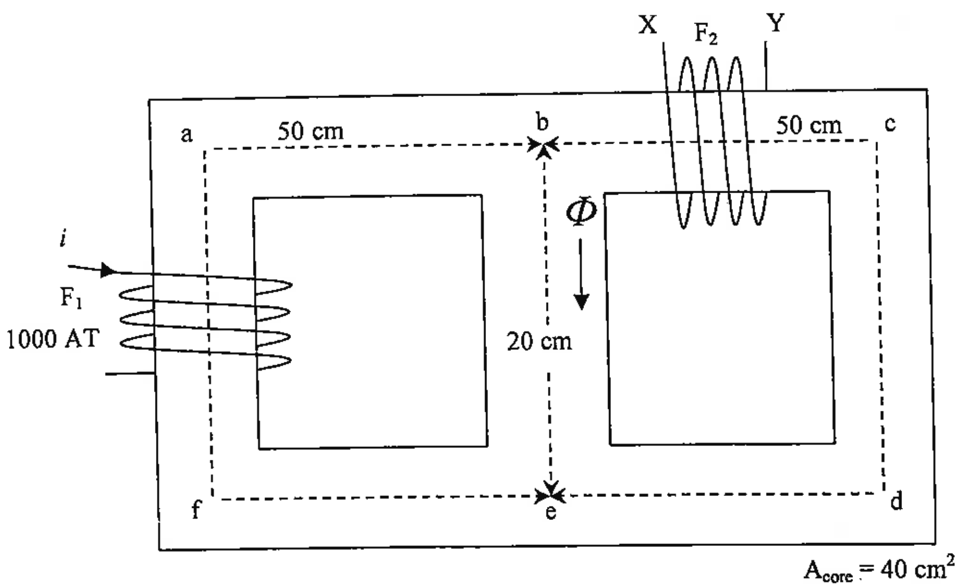

Figure Q1(d) Magnetic circuit

Continued ...

Question 2

a) A vector in spherical coordinate system is given by $\vec{E} = \frac{25}{R^2}\overrightarrow{a_R}$. It is acting on the *x-y* plane ($z = 0$) as shown below:

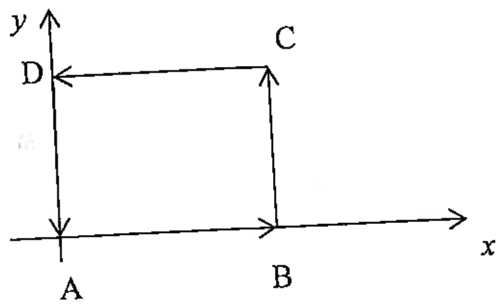

Figure Q2(a) A square on the *x-y* plane ($z = 0$)

Points $A(0,0,0)$, $B(1,0,0)$, $C(1,1,0)$ and $D(0,1,0)$ form a square in anti-clockwise direction as shown in Figure Q2(a).

i) Express the spherical vector, $R\,\overrightarrow{a_R}$ in Cartesian coordinate system involving x, y, z and their unit vectors. Then express $\overrightarrow{a_R}$ in Cartesian system. [3 marks]

ii) Find $\vec{E}$ in Cartesian system at a point with Cartesian coordinates of (1,2,3). [4 marks]

iii) Express $\vec{E}$ in Cartesian system for path A to B. [2 marks]

iv) Express $\vec{E}$ in Cartesian system for path B to C. [2 marks]

v) Express $\vec{E}$ in Cartesian system for path C to D. [1 mark]

vi) The close path integral of $\vec{E}$ around ABCD in anti-clockwise direction is given by $\oint \vec{E}.\overrightarrow{dl}$. Find its value in Cartesian system. [2 marks]

vii) When $\vec{E}$ is electric field with units of V/m, elaborate on the result you obtained in part (vi) above. What is the name of this result in circuit theory? [3 marks]

b) A vector is given as $\vec{B} = (y^2 + 2z)\overrightarrow{a_x} + \left(\frac{\cos z}{x^2+y}\right)\overrightarrow{a_y} + ln(x^2 + y)\overrightarrow{a_z}$.

i) Find the $\overrightarrow{a_x}$ component of the vector $\vec{\nabla} \times \vec{B}$. [4 marks]

ii) Find the $\overrightarrow{a_y}$ component of the vector $\vec{\nabla} \times \vec{B}$. [4 marks]

Continued ...

Question 3

a) A conductor is located from $-L/2$ to $L/2$ along the y-axis and it carries a current I. Point P is located at Cartesian coordinate system of $(x,0,z)$.

i) Sketch the conductor and point P. [3 marks]
ii) Express the current element $d\vec{l}$ in Cartesian system. [2 marks]
iii) Express distant vector from $d\vec{l}$ to P, $\vec{R}$, in Cartesian system. [2 marks]
iv) Express the vector $d\vec{l} \times \vec{R}$ in Cartesian system. [2 marks]
v) Find the magnetic flux density, $\vec{B}$ at point P due to the conductor. [3 marks]

b) A magnetic material with a permeability of μ_1 is interfacing with air. When a magnetic field intensity vector, H_0 passes through this interface, the following is obtained:

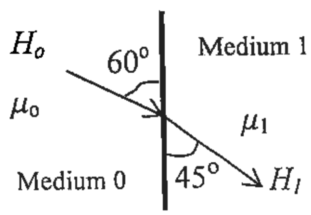

Figure Q3(b) The magnetic field intensity is deflected across the boundary

Find the ratio of μ_1/μ_o . [6 marks]

c) A copper wire is made into a circle of radius r_o with N turns.

i) Find the magnetic flux density, $\vec{B}$ at the centre of the circle. [4 marks]

ii) Find the magnetic energy density, ω_m at the centre of the circle. [3 marks]

Continued ...

Question 4

a) A positive point charge, $+q$, is located at (2,2) on the x-y plane. The x-axis is grounded at 0V. A is a point with Cartesian coordinates of (3,3).

i) Use image method, sketch the two charges and the two distant vectors to point A. [3 marks]

ii) $\vec{r_1}$.is the distant vector from (2,2) to (3,3). Find its unit vector. [2 marks]

iii) $\vec{r_2}$.is the distant vector from (2,-2) to (3,3). Find its unit vector. [2 marks]

iv) Find the total electric field at point A, $\vec{E}$. [4 marks]

b) A spherical capacitor is constructed using two spheres separated by a dielectric medium. The inner sphere has a radius of R_i while the outer sphere has a radius of R_o. The permittivity of the dielectric is ϵ.

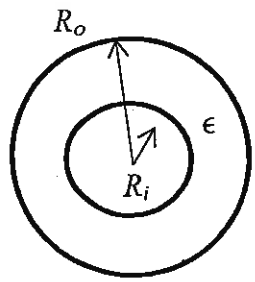

Figure Q4(b) Spherical capacitor with 2 spheres

i) Find the electric field in the dielectric medium, $\vec{E}$ when a total of charge $+Q$ is deposited on the inner sphere. [4 marks]

ii) Find the capacitance of this structure. [6 marks]

c) What is the meaning of 'statics' in electrostatics? [2 marks]

d) Express the two equations from the Maxwell's Equation related to electric charges under magnetostatics condition. [2 marks]

End of Paper

Appendix

Physical Constants, Vector and Coordinate Transformation Relations

Elementary charge	e	1.60×10^{-19} C
Permittivity constant	ε_0	8.85×10^{-12} F/m
Permeability constant	μ_0	1.26×10^{-6} H/m

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}rd\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}Rd\theta + \hat{\phi}R\sin\theta d\phi$
Differential surface areas	$d\vec{s}_x = \hat{x}dydz$ $d\vec{s}_y = \hat{y}dxdz$ $d\vec{s}_z = \hat{z}dxdy$	$d\vec{s}_r = \hat{r}rd\phi dz$ $d\vec{s}_\phi = \hat{\phi}drdz$ $d\vec{s}_z = \hat{z}rdrd\phi$	$d\vec{s}_R = \hat{R}R^2\sin\theta d\theta d\phi$ $d\vec{s}_\theta = \hat{\theta}R\sin\theta dRd\phi$ $d\vec{s}_\phi = \hat{\phi}RdRd\theta$
Differential volume	$dxdydz$	$rdrd\phi dz$	$R^2\sin\theta dRd\theta d\phi$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
1.	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$ $\hat{z} = \hat{z}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
2.	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{r}\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{z}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
3.	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2+y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x}\sin\theta\cos\phi$ $+ \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$ $\hat{\theta} = \hat{x}\cos\theta\cos\phi$ $+ \hat{y}\cos\theta\sin\phi - \hat{z}\sin\theta$ $\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$	$A_R = A_x\sin\theta\cos\phi$ $+ A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi$ $+ A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
4.	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{x} = \hat{R}\sin\theta\cos\phi$ $+ \hat{\theta}\cos\theta\cos\phi - \hat{\phi}\sin\phi$ $\hat{y} = \hat{R}\sin\theta\sin\phi$ $+ \hat{\theta}\cos\theta\sin\phi + \hat{\phi}\cos\phi$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_x = A_R\sin\theta\cos\phi$ $+ A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi$ $+ A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
5.	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r}\sin\theta + \hat{z}\cos\theta$ $\hat{\theta} = \hat{r}\cos\theta - \hat{z}\sin\theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
6.	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{r} = \hat{R}\sin\theta + \hat{\theta}\cos\theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

$(\hat{x} = \vec{a_x}\ ,\ \hat{R} = \vec{a_R}\ ,\ \hat{\theta} = \vec{a_\theta})$

Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r, ϕ, z)

$$\nabla V = \hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi}r & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R, θ, ϕ)

$$\nabla V = \hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{R}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\phi}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$\nabla^2 V = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

Table of Integrals

$$\int \sin^2\theta\, d\theta = \frac{\theta}{2} - \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \cos^2\theta\, d\theta = \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + \text{constant}$$

$$\int \sin\theta\cos^2\theta\, d\theta = \frac{-1}{3}\cos^3\theta + \text{constant}$$

$$\int \cos\theta\sin^4\theta\, d\theta = \frac{1}{5}\sin^5\theta + \text{constant}$$

$$\int \sin 2\theta\, d\theta = \frac{-1}{2}\cos 2\theta + \text{constant}$$

$$\int x\sqrt{a^2 - x^2}\, dx = -\frac{1}{3}\left(a^2 - x^2\right)^{3/2} + \text{constant}$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) + \text{constant}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}}dx = -\sqrt{a^2 - x^2} + \text{constant}$$

$$\int \frac{dz}{\left(r^2 + z^2\right)^{3/2}} = \frac{z}{r^2\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{rdr}{\left(r^2 + z^2\right)^{3/2}} = \frac{-1}{\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + \text{constant}$$

Table of formula

Biot-Savart's Law: $d\vec{B} = \frac{\mu_o I}{4\pi}\frac{d\vec{l}\times\vec{R}}{R^3} = \frac{\mu_o I}{4\pi}\frac{d\vec{l}\times\overrightarrow{a_R}}{R^2}$

Ampere's Law: $\oint \vec{H}\,.\,d\vec{l} = I_{enclosed}$; Magnetic energy density, $\omega_m = \frac{1}{2}\vec{H}.\vec{B}$

Coulomb's Law: $d\vec{E} = \frac{dq}{4\pi\epsilon_o}\frac{\overrightarrow{a_R}}{R^2}$; Electric field: $\vec{E} = -\vec{\nabla}\mathrm{V}$

Gauss's Law: $\oint \vec{E}\,.\,d\vec{s} = Q_{enclosed}/\epsilon$; Electric energy density, $\omega_e = \frac{1}{2}\vec{D}.\vec{E}$

Maxwell's Equations: $\vec{\nabla}.\vec{D} = \rho_v$; $\vec{\nabla}.\vec{B} = 0$; $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$; $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$

Lorentz's Equation: $\vec{F} = q\vec{E} + q\vec{u}\times\vec{B}$

Reluctance: $\Re = \frac{L}{\mu A}$

End of Appendix